Week 6 PYQs
Machine Learning Foundations
May 22 - Sep 25

Note

1. If no questions are listed in that term Quiz, it indicates that the corresponding quiz did not include questions from that week.

2. The answer key may contain minor inaccuracies, as answers were taken directly from the original quiz/exam papers. Some of these might have been corrected later.

3. For any issues or suggestions regarding this document, feel free to contact:
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Quiz 1 May 22

Quiz 2 May 22

1. The matrix

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 1 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix}$$

is

   (a) positive definite

   (b) positive semi-definite

   (c) negative definite

   (d) negative semi-definite

[Ans: a]

2. If $v_1, v_2, \ldots, v_n \in \mathbb{R}^n$ are orthonormal vectors, then which of the following statements is/are true?

   (a) $v_i^T v_j = 0,\ 1 \leq i \leq n,\ 1 \leq j \leq n,\ i \neq j$

   (b) $v_i^T v_i = 1,\ 1 \leq i \leq n$

   (c) Matrix formed by vectors $v_1, \ldots, v_n$ is orthogonal

   (d) Matrix formed by vectors $v_1, \ldots, v_n$ is symmetric

[Ans: A, B, C]

3. Which of the following statements are correct?

   (a) Every positive definite matrix is invertible

   (b) A diagonal matrix with positive entries is positive definite

   (c) A symmetric matrix with positive determinant is not necessarily positive definite

   (d) If matrix S is positive definite then S^{-1} is also positive definite

[Ans: A, B, C, D]

4. The singular value decomposition of matrix

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 3 & 3 & 1 \end{bmatrix}$$

is

(a)

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 4 & 0 & 0 \\ 0 & \sqrt{6} & 0 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{\sqrt{6}} & 0 & \frac{\sqrt{3}}{\sqrt{6}} \\ -\frac{\sqrt{2}}{\sqrt{6}} & -\frac{\sqrt{2}}{\sqrt{6}} & \frac{\sqrt{2}}{\sqrt{6}} \\ \frac{\sqrt{1}}{\sqrt{6}} & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{bmatrix}$$

(b)

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 4 & 0 & 0 \\ 0 & \sqrt{8} & 0 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{\sqrt{6}} & 0 & \frac{\sqrt{3}}{\sqrt{6}} \\ -\frac{\sqrt{2}}{\sqrt{6}} & -\frac{\sqrt{2}}{\sqrt{6}} & \frac{\sqrt{2}}{\sqrt{6}} \\ \frac{\sqrt{1}}{\sqrt{6}} & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{bmatrix}$$

(c)

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 \\ 0 & \sqrt{6} & 0 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{\sqrt{6}} & 0 & \frac{\sqrt{3}}{\sqrt{6}} \\ -\frac{\sqrt{2}}{\sqrt{6}} & -\frac{\sqrt{2}}{\sqrt{6}} & \frac{\sqrt{2}}{\sqrt{6}} \\ \frac{\sqrt{1}}{\sqrt{6}} & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{bmatrix}$$

(d)

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 \\ 0 & \sqrt{8} & 0 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{\sqrt{6}} & 0 & \frac{\sqrt{3}}{\sqrt{6}} \\ -\frac{\sqrt{2}}{\sqrt{6}} & -\frac{\sqrt{2}}{\sqrt{6}} & \frac{\sqrt{2}}{\sqrt{6}} \\ \frac{\sqrt{1}}{\sqrt{6}} & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{bmatrix}$$

[Ans:a]

5. Let matrix

$$A = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$$

Suppose eigenvalues of AA^T are a, b, c. Then find $a + b + c$.

[Ans: 9]

ET May 22

1. Compute the Hessian matrix at the point (0,1,7) of the following 3 variable function: $f(x, y, z) = e^{-x} \cdot \cos(yz)$.

   (a) $\begin{bmatrix} 0 & \pi & 1 \\ \pi & 0 & -1 \\ 1 & 0 & 1 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 & \pi & 1 \\ \pi & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$

   (c) $\begin{bmatrix} 0 & \pi & 1 \\ \pi & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$

   (d) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & \pi^2 & \pi \\ 0 & \pi & 1 \end{bmatrix}$

   (e) None of these

   [Ans: 4]

2. The matrix $A = \begin{bmatrix} 4 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix}$ is:

   (a) positive definite

   (b) positive semi-definite

   (c) negative definite

   (d) negative semi-definite

   (e) None of these

   [Ans: 1]

Quiz 1 Sep 22

Quiz 2 Sep 22

1. Consider the matrix
$$A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$
and choose the correct option(s):

   (a) The eigenvalues of A are non-negative

   (b) The matrix A is a positive semidefinite matrix

   (c) The eigenvectors of A are $v_1 = (-1, 1), v_2 = (1, 2)$

   (d) The matrix A is positive definite

   [Ans: A, B]

2. Let v_1 and v_2 be two non-zero real vectors of size 3×1. Suppose that $v_1^T v_2 = 0$, $v_1^T v_1 = 1$ and $v_2^T v_2 = 1$. Let A be a 3×3 matrix given as
$$A = 2v_1 v_1^T + 3v_2 v_2^T,$$
then eigenvalues of A are λ_1, λ_2 and λ_3. Find the value of $\lambda_1 + \lambda_2 + \lambda_3$. [Ans:5]

3. Let
$$A = \begin{bmatrix} 2 & -2 & 0 \\ -2 & 1 & -2 \\ 0 & -2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 2 & -2 \\ 2 & 5 & -4 \\ -2 & -4 & 5 \end{bmatrix}$$
Choose the correct option:

   (a) Only A is positive definite

   (b) Only B is positive definite

   (c) Both A and B are positive definite

   (d) None is positive definite

   [Ans: b]

4. The SVD of matrix
$$A = \begin{bmatrix} -4 & -7 \\ 1 & 4 \end{bmatrix}$$
is

   (a)
$$\begin{bmatrix} -0.894 & 0.447 \\ 0.447 & 0.894 \end{bmatrix} \begin{bmatrix} \sqrt{81} & 0 \\ 0 & \sqrt{1} \end{bmatrix} \begin{bmatrix} 0.447 & 0.894 \\ -0.894 & 0.447 \end{bmatrix}$$

(b)

$$\begin{bmatrix} -0.447 & 0.894 \\ -0.894 & -0.447 \end{bmatrix} \begin{bmatrix} \sqrt{40} & 0 \\ 0 & \sqrt{10} \end{bmatrix} \begin{bmatrix} -0.707 & 0.707 \\ 0.707 & 0.707 \end{bmatrix}$$

(c)

$$\begin{bmatrix} -0.707 & 0.707 \\ 0.707 & 0.707 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & -5 \end{bmatrix} \begin{bmatrix} -0.447 & 0.894 \\ -0.894 & -0.447 \end{bmatrix}$$

(d)

$$\begin{bmatrix} -0.512 & 0.707 \\ 0.707 & -0.512 \end{bmatrix} \begin{bmatrix} \sqrt{40} & 0 \\ 0 & \sqrt{10} \end{bmatrix} \begin{bmatrix} 0.707 & -0.707 \\ 0.707 & 0.707 \end{bmatrix}$$

[Ans:a]

ET Sep 22

1. For what value of k, the function $f(x) = k{x_1}^2 + {x_2}^2 - kx_1x_2$ is positive semi-definite?

   (a) 0

   (b) 1

   (c) 4

   (d) 2

   [Ans: 3]

2. Let f be a function of three variables. The eigenvalues of Hessian of f are 2, 4, 1. Then what does f give?

   (a) Minima.

   (b) Maxima.

   (c) Saddle.

   (d) It can be either maxima or minima.

   [Ans: 1]

Quiz 1 Jan 23

Quiz 2 Jan 23

1. Find the rank one approximation of the matrix

$$A = \begin{bmatrix} 2 & 0 \\ 0 & -3 \\ 0 & 0 \end{bmatrix}$$

corresponding to its largest eigenvalue.

(a)
$$\begin{bmatrix} 4 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

(b)
$$\begin{bmatrix} 0 & 0 \\ 0 & 3 \\ 0 & 0 \end{bmatrix}$$

(c)
$$\begin{bmatrix} 0 & 0 \\ 0 & -9 \\ 0 & 0 \end{bmatrix}$$

(d)
$$\begin{bmatrix} 0 & 0 \\ 0 & -3 \\ 0 & 0 \end{bmatrix}$$

[Ans:d]

2. Let A be an $n \times n$ positive definite matrix. Then which among the following statements are correct?

   (a) A^{-1} is positive definite
   (b) $A + B$ is positive definite, if B is positive definite
   (c) Rank$(A) = n - 1$
   (d) A^2 is positive definite

[Ans: a, b ,d]

3. Consider a matrix

$$A = \begin{bmatrix} 2 & b \\ b & 8 \end{bmatrix}$$

For what value of b is the matrix A positive definite?

(a) $b < 4$

(b) $b > -4$

(c) $b > 4$ and $b < -4$

(d) $-4 < b < 4$

[Ans:d]

4. Find the minimum value of

$$\frac{1}{2}(2x^2 + 2bxy + 8y^2) - x$$

for b in the valid range based on previous question

(a) $\frac{4}{b^2-16}$

(b) $-\frac{4}{b^2-16}$

(c) $\frac{8}{b^2-16}$

(d) $-\frac{8}{b^2-16}$

[Ans:a]

ET Jan 23

1. Suppose A is a symmetric positive definite matrix and Q is a orthogonal matrix, then which of the following statements is/are true?

   (a) $Q^T AQ$ is a diagonal matrix.

   (b) $Q^T AQ$ is symmetric positive definite.

   (c) $Q^T AQ$ has the same eigenvalues as A.

   (d) e^{-A} is symmetric positive definite.

[Ans: 2, 3, 4]

Quiz 1 May 23

Quiz 2 May 23

1. Which among the following statements is/are true?

   (a) Positive semidefinite function has only global minimum
   (b) Positive semidefinite function has both min and max
   (c) Negative semidefinite function has only global maximum
   (d) Negative semidefinite function has both min and max

   [Ans: A, C]

2. Which among the following functions are positive definite?

   (a) $Q(x, y) = x^2 - xy + y^2$
   (b) $Q(x, y) = x^2 - 2xy + y^2$
   (c) $Q(x, y) = x^2 + xy$

   [Ans: A]

3. Given the following information about a 4×2 matrix A:

   - The characteristic polynomial of $A^T A$ is $(\lambda - 48)(\lambda - 12)$.
   - Eigenvectors of $A^T A$ corresponding to eigenvalues $\lambda = 48, \lambda = 12$ are $q_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $q_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.
   - $$Aq_1 = \begin{bmatrix} 4/\sqrt{2} \\ -8/\sqrt{2} \\ 0 \\ 4/\sqrt{2} \end{bmatrix}, \quad Aq_2 = \begin{bmatrix} -2/\sqrt{2} \\ 0 \\ 4/\sqrt{2} \\ 2/\sqrt{2} \end{bmatrix}$$

   What is the matrix A?

   (a)
   $$\begin{bmatrix} 1 & 3 \\ 0 & 0 \\ 2 & -2 \\ 1 & -1 \end{bmatrix}$$

   (b)
   $$\begin{bmatrix} 2 & 2 \\ -4 & -4 \\ 0 & 0 \\ 2 & 2 \end{bmatrix}$$

(c)

$$\begin{bmatrix} -1 & 1 \\ 0 & 0 \\ 2 & -2 \\ 1 & -1 \end{bmatrix}$$

(d)

$$\begin{bmatrix} 1 & 3 \\ -4 & -4 \\ 2 & -2 \\ 3 & 1 \end{bmatrix}$$

[Ans: 4]

ET May 23

1. Let A be an $n \times d$ matrix with right-singular vectors $v_1, ..., v_r$, left-singular vectors $u_1, ..., u_r$, and corresponding singular values $\sigma_1, \sigma_2, ..., \sigma_r$ If $A = \sum_{i=1}^{r} \sigma_i u_i v_i^T$, select the correct options from the following:

   (a) Each of $\sigma_i u_i v_i^T$ is a rank one matrix.

   (b) Each of $\sigma_i u_i v_i^T$ is a matrix of rank r.

   (c) For $B = A^T A$, $B = \sum_{i=1}^{r} \sigma_i^2 v_i v_i^T$.

   (d) For $B = AA^T$, $B = \sum_{i=1}^{r} \sigma_i^2 v_i v_i^T$.

[Ans: 1, 3]

Quiz 1 Sep 23

Quiz 2 Sep 23

1. Let

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

One of the eigenvalue of A is given to be 1 with the corresponding eigenvector $[0,1,0]^T$. Find the Schur's decomposition of A, i.e., find a matrix U with orthonormal columns such that $U^T AU$ is uppertriangular.

(a)
$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(b)
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(c)
$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(d)
$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

[Ans: c]

2. Consider

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

(a) Find the condition number $\frac{\sigma_1}{\sigma_3}$.

[Ans: 2.23]

(b) Is the matrix A positive definite?

i. Yes

ii. No

[Ans: 1]

(c) Suppose the SVD of A is given. What is the rank of A?

[Ans: 2]

(d) Which of the following options are true?

i. $\text{Columnspace}(A) = \text{span}\left\{ \begin{bmatrix} 4 \\ 5 \\ -2 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix} \right\}$

ii. $\text{Columnspace}(A) = \text{span}\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} \right\}$

iii. $\text{Nullspace}(A) = \text{span}\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} \right\}$

iv. $\text{Nullspace}(A) = \text{span}\left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

[Ans: 1, 4]

ET Sep 23

1. Let A be an $n \times n$ positive definite matrix. Then, which among the following are correct?

   (a) A^k is positive definite, for all $k \geq 1$.

   (b) Each of the diagonal entry of A will be positive.

   (c) rA is positive definite, $r \in \mathbb{R}$

   (d) A^{-1} is positive definite.

[Ans: 1, 2, 4]

Quiz 1 Jan 24

Quiz 2 Jan 24

1. Consider
$$A = \begin{bmatrix} 2 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(a) Let $A = Q_1 \Sigma Q_2^T$ be the SVD of A. Which of the following options is/are true?

i. $A^T A = \begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}$

ii. Eigenvalues of $A^T A$ are 6 and 2

iii. An eigenvector corresponding to eigenvalue 6 is $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

iv. Orthonormalized eigenvectors of $A^T A$ are $\begin{bmatrix} \frac{2\sqrt{5}}{5} \\ \frac{\sqrt{5}}{5} \end{bmatrix}, \begin{bmatrix} -\frac{\sqrt{5}}{5} \\ \frac{2\sqrt{5}}{5} \end{bmatrix}$

[Ans: 3, 4]

(b) Which of the following options is/are true?

i. $\Sigma = \begin{bmatrix} \sqrt{6} & 0 \\ 0 & \sqrt{2} \\ 0 & 0 \end{bmatrix}$

ii. $\Sigma = \begin{bmatrix} \sqrt{6} & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

iii.
$$Q_2 = \begin{bmatrix} \frac{2\sqrt{5}}{5} & -\frac{\sqrt{5}}{5} \\ \frac{\sqrt{5}}{5} & \frac{2\sqrt{5}}{5} \end{bmatrix}$$

iv.
$$Q_2 = \begin{bmatrix} \frac{2\sqrt{5}}{5} & \frac{\sqrt{5}}{5} \\ \frac{\sqrt{5}}{5} & \frac{2\sqrt{5}}{5} \end{bmatrix}$$

[Ans: 2, 3]

2. Consider
$$f(x_1, x_2) = 2x_1^2 + 12x_1x_2 + 7x_2^2 = (x_1 \;\; x_2) A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(a) Which of the following represents matrix A?

i. $\begin{bmatrix} 2 & 6 \\ 6 & 7 \end{bmatrix}$

ii. $\begin{bmatrix} 2 & -6 \\ -6 & 7 \end{bmatrix}$

iii. $\begin{bmatrix} -2 & 6 \\ 6 & -7 \end{bmatrix}$

iv. $\begin{bmatrix} -2 & -6 \\ -6 & -7 \end{bmatrix}$

[Ans: 1]

(b) Which of the following options is/are true?

i. A is a negative definite matrix

ii. A is a positive definite matrix

iii. A is a negative semi-definite matrix

iv. A is a positive semi-definite matrix

v. A is an indefinite matrix

[Ans: 5]

ET Jan 24

1. Let A be a real symmetric positive definite matrix. Which of the following options is/are true?

   (a) The eigenvectors of A may or may not be orthogonal.

   (b) If A is similar to a diagonal matrix B, then B is also positive definite.

   (c) A^TA is not positive definite.

   (d) A^TA is not similar to a diagonal matrix.

[Ans: 2]

Quiz 1 May 24

Quiz 2 May 24

1. Consider

$$f(x_1, x_2, x_3) = (x_1\ x_2\ x_3)A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 2x_1x_2 + 4x_2x_3 - x_1^2 - 5x_2^2 - 3x_3^2$$

   (a) Which of the following represents matrix A?

   i. $\begin{bmatrix} 1 & 1 & 0 \\ 1 & -5 & 2 \\ 0 & 2 & -3 \end{bmatrix}$

   ii. $\begin{bmatrix} -1 & -1 & 0 \\ 1 & -5 & 2 \\ 0 & 2 & -3 \end{bmatrix}$

   iii. $\begin{bmatrix} -1 & 1 & 0 \\ 1 & -5 & 2 \\ 0 & 2 & -3 \end{bmatrix}$

   iv. $\begin{bmatrix} 1 & -1 & 0 \\ 1 & -5 & 2 \\ 0 & 2 & -3 \end{bmatrix}$

[Ans: 3]

   (b) The matrix A is

   i. positive definite

   ii. negative definite

[Ans: 2]

2. Consider

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix}$$

   (a) Which of the following is correct for Q_1?

   i. $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

   ii. $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

   iii. $\frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

   iv. $\sqrt{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

[Ans: 2]

(b) Which of the following is correct for Q_2?

i. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

ii. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

iii. $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

iv. $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

[Ans: 1]

(c) Which of the following is correct for Σ?

i. $\begin{bmatrix} \sqrt{2} & 0 & 0 \\ 0 & -\sqrt{2} & 0 \end{bmatrix}$

ii. $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix}$

iii. $\begin{bmatrix} -\sqrt{2} & 0 & 0 \\ 0 & -\sqrt{2} & 0 \end{bmatrix}$

iv. $\begin{bmatrix} \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{bmatrix}$

[Ans: 4]

3. Which of the following options is/are true?

(a) Any matrix of order $n \times m$ is similar to an upper triangular matrix

(b) If A is not symmetric, then A^TA cannot be symmetric

(c) If A is positive definite, then A^2 is positive definite

(d) For any matrix A, A^TA is orthogonally diagonalizable

[Ans: 3, 4]

ET May 24

Quiz 1 Sep 24

Quiz 2 Sep 24

1. Consider
$$A = \begin{bmatrix} 2 & -i & i \\ i & 2 & -i \\ -i & i & 2 \end{bmatrix}$$
Diagonalize as $A = Q\Lambda Q^T$. Then Λ is

(a) $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3+\sqrt{2} & 0 \\ 0 & 0 & 3-\sqrt{2} \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2+\sqrt{3} & 0 \\ 0 & 0 & 2-\sqrt{3} \end{bmatrix}$

(c) $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2+\sqrt{3} & 0 \\ 0 & 0 & 2-\sqrt{3} \end{bmatrix}$

(d) Not diagonalizable

[Ans: 2]

2. Given
$$Q(x, y, z) = 3x^2 + 2y^2 + z^2 + 4xy + 2xz + 4yz$$
expressed as $[x\ y\ z]M[x\ y\ z]^T$, find M.

(a) $\begin{bmatrix} 3 & 2 & 1 \\ 2 & 2 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 3 & 1 \\ 3 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(d) None

[Ans: 1]

3. Consider the matrix $A = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \end{pmatrix}$. Let the singular value decomposition (SVD) form of A be written as $A = Q_1 \Sigma Q_2^T$.

(a) Which of the following options is/are true?

i. $Q_1 = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

ii. $Q_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

iii. $Q_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$

iv. $Q_1 = \sqrt{2} \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}$

[Ans: 2]

(b) Which of the following options is/are true?

i. $Q_2^T = \frac{1}{\sqrt{6}} \begin{pmatrix} 2 & -1 & 1 \\ 0 & \sqrt{3} & \sqrt{3} \\ -\sqrt{2} & 1 & \sqrt{2} \end{pmatrix}$

ii. $Q_2^T = \frac{1}{\sqrt{6}} \begin{pmatrix} 2 & -1 & 1 \\ 0 & \sqrt{3} & \sqrt{3} \\ -\sqrt{2} & -\sqrt{2} & -\sqrt{2} \end{pmatrix}$

iii. $Q_2^T = \frac{1}{\sqrt{6}} \begin{pmatrix} 2 & -1 & 1 \\ 0 & \sqrt{3} & \sqrt{3} \\ \sqrt{2} & -\sqrt{2} & \sqrt{2} \end{pmatrix}$

iv. $Q_2^T = \frac{1}{\sqrt{6}} \begin{pmatrix} 2 & -1 & 1 \\ 0 & \sqrt{3} & \sqrt{3} \\ -\sqrt{2} & -\sqrt{2} & \sqrt{2} \end{pmatrix}$

[Ans: 4]

(c) Which of the following options is true?

i. $\Sigma = \begin{pmatrix} \sqrt{3} & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

ii. $\Sigma = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

iii. $\Sigma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

iv. $\Sigma = \begin{pmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{3} & 0 \end{pmatrix}$

[Ans: 1]

4. Which of the following statements is true about the matrix $A = \begin{pmatrix} 1 & 4 \\ 4 & 16 \end{pmatrix}$?

(a) A is positive definite.

(b) A is positive semi-definite.

(c) A is negative definite.

(d) A is negative semi-definite.

[Ans: 2]

ET Sep 24

1. Consider the quadratic form $f(x_1, x_2, x_3) = \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 2x_1x_2 + x_1^2 + 5x_2^2 + 3x_3^2$ where A is a matrix. Which of the following is/are true for matrix A?

   (a) A is a positive definite matrix.

   (b) A is not a singular matrix.

   (c) A is not a symmetric matrix.

   (d) A is a negative definite matrix.

   [Ans: 1, 2]

Quiz 1 Jan 25

Quiz 2 Jan 25

1. Let A be an $n \times n$ positive definite matrix with singular values $\sigma_1, \sigma_2, ..., \sigma_n$. Which of the following is true?

   (a) $\sigma_i \geq 0$ for all i.

   (b) A has at least one zero singular value.

   (c) The eigenvalues of A must be negative

   (d) If $\sigma_i = 0$ for all i, then A is the zero matrix.

   [Ans: 1, 4]

2. Consider the matrix $M = \begin{bmatrix} 2 & 1 & 0 \\ 1 & -2 & 0 \end{bmatrix}$. Let the singular value decomposition (SVD) form of M be written as $M = Q_1 \Sigma Q_2^T$. Which of the following options is/are true for Q_1?

   (a) $Q_1 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

   (b) $Q_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

   (c) $Q_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$

   (d) $Q_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

   [Ans: 4]

3. Consider the matrix $M = \begin{bmatrix} 2 & 1 & 0 \\ 1 & -2 & 0 \end{bmatrix}$. Which of the following options is/are true for Q_2^T?

   (a) $Q_2^T = \frac{1}{\sqrt{6}} \begin{bmatrix} 3 & -1 & 1 \\ 0 & \sqrt{3} & \sqrt{3} \\ -\sqrt{2} & 1 & \sqrt{2} \end{bmatrix}$

   (b) $Q_2^T = \begin{bmatrix} 3 & -1 & 1 \\ 0 & \sqrt{3} & \sqrt{3} \\ -\sqrt{2} & -\sqrt{2} & -\sqrt{2} \end{bmatrix}$

   (c) $Q_2^T = \frac{1}{\sqrt{6}} \begin{bmatrix} 3 & -1 & 1 \\ 0 & \sqrt{3} & \sqrt{3} \\ \sqrt{2} & -\sqrt{2} & \sqrt{2} \end{bmatrix}$

(d) $Q_2^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

[Ans: 1, 3]

ET Jan 25

1. Which of the following statements about singular value decomposition (SVD) is/are true?

   (a) Singular value decomposition is possible only for a square matrix.

   (b) Singular value decomposition is possible only for a symmetric matrix.

   (c) The singular values of a matrix are always non-negative.

   (d) The singular values of matrices A and A^T are the same.

[Ans: 3, 4]

Quiz 1 May 25

Quiz 2 May 25

1. Given the quadratic form $Q(x, y, z) = x^2 + 2xy - 2xz$. Which of the following is true?

   (a) Q is positive definite.

   (b) Q is positive semi-definite.

   (c) Q is negative definite.

   (d) Q is negative semi-definite.

   (e) None of these

   [Ans: 5]

2. Which of the following statements is true about the matrix $A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$?

   (a) A is positive definite.

   (b) A is positive semi-definite.

   (c) A is negative definite.

   (d) A is negative semi-definite.

   [Ans: 1]

3. Let $A \in \mathbb{R}^{n \times d}$ be a centered data matrix, where $d > n$. Which of the following statements about the eigenvalues of $AA^T \in \mathbb{R}^{n \times n}$ and $A^TA \in \mathbb{R}^{d \times d}$ is true?

   (a) AA^T and A^TA have different sets of non-zero eigenvalues.

   (b) The non-zero eigenvalues of AA^T and A^TA are equal.

   (c) All eigenvalues of A^TA are zero.

   (d) A^TA has more non-zero eigenvalues than AA^T.

   [Ans: 2]

ET May 25

Quiz 1 Sep 25

Quiz 2 Sep 25

1. Consider the quadratic form $f(x, y, z) = x^2 - y^2 + 2z^2 + 2xy - 4yz + 4xz$. If $f(x, y, z) = v^T Av$ where $v = (x, y, z)^T$ and A is a 3×3 matrix, which of the following is A?

   (a) $\begin{bmatrix} 1 & 1 & 2 \\ 1 & -1 & -2 \\ 2 & -2 & 2 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 & 2 & 2 \\ 0 & -1 & -1 \\ 2 & -2 & 2 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & -4 \\ 2 & -2 & 2 \end{bmatrix}$

   (d) $\begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & -2 \\ 1 & -1 & 1 \end{bmatrix}$

   [Ans: 1]

2. Consider the quadratic form $f(x, y, z) = x^2 - y^2 + 2z^2 + 2xy - 4yz + 4xz$. If $f(x, y, z) = v^T Av$ where $v = (x, y, z)^T$ and A is a 3×3 matrix, which of the following is A?

   (a) $\begin{bmatrix} 1 & 1 & 2 \\ 1 & -1 & -2 \\ 2 & -2 & 2 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 & 2 & 2 \\ 0 & -1 & -1 \\ 2 & -2 & 2 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & -4 \\ 2 & -2 & 2 \end{bmatrix}$

   (d) $\begin{bmatrix} 1 & 3 & 2 \\ 1 & -1 & -2 \\ 1 & -1 & 1 \end{bmatrix}$

   [Ans: 1]

3. Consider a matrix A of shape 2×2. The SVD of A is given by $U\Sigma V^T$. The columns of U are the left singular vectors and the columns of V are the right singular vectors. The following details are given:

   - The first left singular vector is $u = (\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}})$

- The first right singular vector is $v = (\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}})$
- The first singular value is $\sigma = 5$

Find $v^T Av$. [Ans: 3.9 to 4.1]

4. The singular value decomposition exists only for square matrices.

   (a) True

   (b) False

   [Ans: 2]

5. Which of the following are the singular values of $A = \frac{1}{4}\begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$?

   (a) $1, \frac{1}{2}$

   (b) $1, 2$

   (c) $3, 0$

   (d) $2, \frac{-1}{2}$

   [Ans: 1]

6. If B is a symmetric matrix, consider statements P and Q:

   - P: If λ is an eigenvalue of B, then $|\lambda|$ is a singular value of B.
   - Q: If λ is a singular value of B, then λ is an eigenvalue of B.

   Select the most appropriate option.

   (a) P is true, Q is false

   (b) Q is true, P is false

   (c) Both P and Q are true

   (d) Both P and Q are false

   [Ans: 1]

ET Sep 25

1. Let $f(x, y) = x^2y + y^3 - 2x^2 - 3y^2 + 1$. What is the Hessian matrix $H(x, y)$ of the function $f(x, y)$?

   (a) $\begin{pmatrix} 2y-4 & 2x \\ 2x & 6y-6 \end{pmatrix}$

   (b) $\begin{pmatrix} 2x & 2y-4 \\ 6y-6 & 2x \end{pmatrix}$

   (c) $\begin{pmatrix} 6y-6 & 2x \\ 2x & 2y-4 \end{pmatrix}$

   (d) $\begin{pmatrix} 2y-4 & 6y-6 \\ 2x & 2x \end{pmatrix}$

   [Ans: 1]

2. Let A be an $m \times n$ matrix. The Singular Value Decomposition (SVD) of A is $A = U\Sigma V^T$. Which of the following is true?

   (a) U and V are orthogonal matrices.

   (b) Σ is a diagonal matrix containing singular values.

   (c) The singular values are the square roots of the eigenvalues of A^TA.

   (d) All of the above.

   [Ans: 4]